

Conventional Implicature as a scope phenomenon

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Goals for today

- ▶ Give a semantics for non-restrictive relatives, where supplemental content interacts with its semantic context by **taking scope**.
- ▶ Concretely, I'll extend the dynamic semantics of Charlow 2014 with some apparatus for two-dimensional content, à la Potts 2005.
- ▶ I argue that the extension is in fact **all we need to say** – the empirical properties we're after just fall out.

Where we are

Data

Composition: dynamic side effects

Adding in CIs

Accounting for our data

Wrapping up

Basic data

- ▶ Today we'll be looking at non-restrictive relative clauses (**NRRCs**):

(1) Sue, who's smart, bribed Jon, who isn't.

- ▶ Two related questions:
 - ▶ What sort of meaning should we assign this sentence?
 - ▶ How is the NRRC compositionally integrated?

Non-interaction

- ▶ In general, content introduced by NRRCs seems not to interact with other operators in a sentence:
 - (2) I didn't read *Beowulf*, which is a stone-cold classic.
 - (3) If John, who likes dancing, comes, the party will be great.
- ▶ Reminiscent of presupposition, but ultimately distinct (e.g., as Potts 2005 points out, the NRRC's content can't be presupposed!).

Scope of the anchor

- ▶ Scope of the anchor (after AnderBois et al. 2015: ex.72):
 - (4) John didn't read a book, which Mary had recommended.
- ▶ This sentence **only** allows a wide-scope reading for the indefinite.

Quantifiers not welcome

- ▶ Quantifiers cannot serve as anchors:

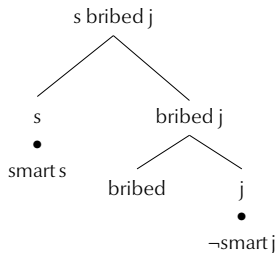
(5) I read {*Beowulf*, a book, *no book}, which Mary likes.

Binding

- ▶ An indefinite can bind out of a NRRC (e.g. AnderBois et al. 2015):
 - (6) John, who nearly killed a woman_i with his car, visited her_i in the hospital.
- ▶ In fact, binding can go both ways:
 - (7) A boy_i read Beowulf, which he_i loved.
- ▶ Quantifiers, alas, still not allowed:
 - (8) *John, who nearly killed no woman_i with his car, visited her_i in the hospital.
 - (9) *No boy_i read Beowulf, which he_i loved.
- ▶ Surprising: we see the NRRC semantically interacting with the rest of the sentence, despite apparent non-interaction observed prior.

Potts 2005: radical separation

- ▶ Potts-style **parsetree** for *Sue, who's smart, bribed Jon, who isn't*:



- ▶ This representation is interpreted by pruning the bulleted meanings and conjoining them in a separate dimension.
- ▶ Worries: non-compositional (à la e.g. DRT), how to get interaction between the two dimensions? What is special about indefinites?

AnderBois et al. 2015: no separation

- ▶ No distinguished dimension for supplemental content.
- ▶ Distinguish **two kinds of updates**:
 - ▶ Proposals to update the common ground, subject to negotiation.
 - ▶ Immediate, non-negotiated updates to the common ground.
- ▶ Proposals associated with at-issue content, impositions with not-at-issue content (e.g. what's introduced by a NRRC).
- ▶ Worry: many of the features that fall out of a two-dimensional analysis need to be stipulated:
 - ▶ Non-interaction
 - ▶ Types of anchors
 - ▶ Scope of the anchor
 - ▶ Differential binding capabilities of indefinites, true quantifiers

Upgrading a dynamic semantics?

- ▶ Something you might hope for: start with a dynamic semantics, tack on a second dimension, and let the chips fall where they may.

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Example: nondeterminism

- ▶ It is sometimes useful to entertain multiple values in parallel:

$$\llbracket \text{a linguist} \rrbracket = \{x \mid \text{ling } x\}$$

$$\llbracket \text{John met a linguist} \rrbracket = \{j \text{ met } x \mid \text{ling } x\}$$

- ▶ Usual approach is to enrich composition to handle sets:

$$\llbracket A B \rrbracket = \{f x \mid f \in \llbracket A \rrbracket \wedge x \in \llbracket B \rrbracket\}$$

- ▶ Another, equally valid possibility is to suppose that alternatives *take scope* (Charlow 2015).

Scoping alternatives

- ▶ Requires **two familiar type-shifters**.
- ▶ First: `return` is Karttunen 1977's C_o , aka Partee 1986's `IDENT`. It turns a boring thing into a fancy thing (though still fairly boring).

$$\text{return } x := \{x\}$$

- ▶ Second: \gg turns a set m into a scope-taker by feeding each member of m to a scope κ and unioning the resulting sets.

$$m \gg \kappa := \bigcup_{x \in m} \kappa x$$

- ▶ E.g., $\{x \mid \text{linguist } x\} \gg = \lambda \kappa. \bigcup_{\text{linguist } x} \kappa x$.¹

¹ $\{x \mid \text{linguist } x\} \gg$ is actually equivalent to the meaning Cresti 1995 assigns to *which linguist*, and also crops up in Heim 2000; Ciardelli & Roelofsen to appear.

Fancy, boring types

- ▶ Typing judgments, where Fa should be read as “a fancy a ”. In this case, a fancy a is simply a set of a 's, so $Fa ::= \{a\} ::= a \rightarrow t$:

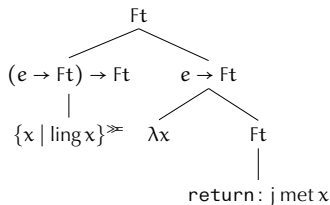
$$\text{return} :: a \rightarrow Fa \quad (\gg) :: Fa \rightarrow (a \rightarrow Fb) \rightarrow Fb$$

- ▶ `return` and `gg` build a bridge between fancy things (sets of alternatives) and boring things (familiar denotations):

$$\underbrace{m}_{(a \rightarrow Fb) \rightarrow Fb} \gg \overbrace{(\lambda x. \text{return} \dots x \dots)}^{a \rightarrow Fb}$$

An example

- ▶ An example of how this works for *John met a linguist*:



- ▶ Gives the expected set of propositions, about different linguists:

$$\{j \text{ met } x \mid \text{ling } x\}$$

- ▶ This pattern will be repeated time and again. The alternative generator takes scope via (\gg), `return` applies to its remnant.

State-sensitivity: the Reader monad

- ▶ Some things (e.g., pronouns) are sensitive to the state at which they're evaluated. Suggests the following fancy type:

$$\mathbf{F}a ::= s \rightarrow a$$

- ▶ Along with the following monadic operations:

$$\text{return } x := \lambda i. x \qquad \mathbf{m} \gg \kappa := \lambda i. \kappa (\mathbf{m} \ i) \ i$$

- ▶ Example, supposing $\mathbf{SHE}_0 := \lambda i. i_0$

$$\mathbf{SHE}_0^{\gg} (\lambda x. \text{return } : x \ \text{left}) = \lambda i. i_0 \ \text{left}$$

Combining the two: the Reader + Set monad

- ▶ We might even combine nondeterminism and state-sensitivity, taking fancy \mathbf{a} 's to be **functions from states** into sets of \mathbf{a} 's.

$$F\mathbf{a} = s \rightarrow \{\mathbf{a}\}$$

- ▶ This in turn implies minimally tweaked versions of `return` and \gg :

$$\text{return } x := \lambda i. \{x\} \qquad m \gg \kappa := \lambda i. \bigcup_{x \in m i} \kappa x i$$

- ▶ Example, supposing $\mathbf{A.LING} := \lambda i. \{x \mid \text{ling } x\}$ and $\mathbf{HER}_0 := \lambda i. \{i_0\}$.

$$\begin{aligned} \mathbf{A.LING} \gg (\lambda x. \mathbf{HER}_0 \gg (\lambda y. \text{return} : x \text{ met } y)) \\ = \lambda i. \{x \text{ met } i_0 \mid \text{ling } x\} \end{aligned}$$

The Monad Slide

- ▶ Any return, (\gg) decomposes LIFT (e.g. Partee 1986):

$$(\text{return } x) \gg = \text{LIFT } x = \lambda \kappa. \kappa x$$

- ▶ They also form something known in category theory & computer science as a **monad** (e.g. Moggi 1989; Wadler 1992, 1995).
 - ▶ In general, monads are *really* good at allowing (arbitrarily) fancy things to interact with boring things.
 - ▶ See Shan 2002; Giorgolo & Asudeh 2012; Unger 2012; Charlow 2014 for discussions of monads in natural language semantics.

do-notation

- ▶ There's a convenient notation for working with these "LFs":

$$\begin{array}{l} \text{do } x \leftarrow m \\ \quad y \leftarrow n \\ \quad \quad \vdots \\ \quad \text{return: } \phi \end{array} = m \gg= (\lambda x. n \gg= (\lambda y. \dots \text{return: } \phi))$$

- ▶ Standard sugaring in Haskell, essentially the "monad comprehensions" of Wadler 1992.

Examples of do-notation

- ▶ An example with alternatives (using the Set monad):

$$\begin{array}{l} \text{do } x \leftarrow \{x \mid \text{ling } x\} \\ \text{return: } j \text{ met } x \end{array} = \{j \text{ met } x \mid \text{ling } x\}$$

- ▶ An example with state-sensitivity (using the Reader monad):

$$\begin{array}{l} \text{do } x \leftarrow \lambda i. i_0 \\ \text{return: } j \text{ met } x \end{array} = \lambda i. j \text{ met } i_0$$

- ▶ And an example with both (using the Reader + Set monad):

$$\begin{array}{l} \text{do } x \leftarrow \lambda i. \{x \mid \text{ling } x\} \\ \text{y} \leftarrow \lambda i. \{i_0\} \\ \text{return: } x \text{ met } y \end{array} = \lambda i. \{x \text{ met } i_0 \mid \text{ling } x\}$$

Dynamics: basic data

- ▶ A familiar data point: Indefinites behave more like names than quantifiers with respect to anaphoric phenomena.

(10) {Polly_i, a linguist_i, *every linguist_i} came in. She_i sat.

Discourse referents

- ▶ Dynamic semantics: sentences add discourse referents to the “conversational scoreboard” (e.g. Groenendijk & Stokhof 1991):

$$i \longrightarrow \llbracket \text{Polly came in} \rrbracket \longrightarrow i + p$$

- ▶ Indefinites (but not quantifiers) also set up discourse referents. In case four linguists came in – a, b, c, and d – we’ll have:

$$i \longrightarrow \llbracket \text{a linguist came in} \rrbracket \begin{cases} \longrightarrow i + a \\ \longrightarrow i + b \\ \longrightarrow i + c \\ \longrightarrow i + d \end{cases}$$

- ▶ Formally captured by modeling meanings as relations on states. For example, here is a candidate meaning for *a linguist came in*:

$$\lambda i. \{i + x \mid \text{linguist } x \wedge \text{came } x\}$$

Folding in dynamics

- ▶ It's straightforward to fold dynamics into the monadic perspective.
- ▶ Dynamics relies on the ability to output modified assignments (indeed, given indefinites, to output *alternative* assignments).
- ▶ One way to think of this is in terms of a new “fancy” type:

$$F\mathbf{a} ::= s \rightarrow \{\langle \mathbf{a}, s \rangle\}$$

- ▶ An upgrade from the previous semantics, where $F\mathbf{a} ::= s \rightarrow \{\mathbf{a}\}$.
- ▶ The monadic operations again essentially follow from the types:

$$\text{return: } x := \lambda i. \{\langle x, i \rangle\}$$

$$m \ggg \kappa := \lambda i. \bigcup_{\langle x, j \rangle \in m i} \kappa x j$$

Basic meanings

- ▶ Meaning for an indefinite:

$$\mathbf{A.LING} = \lambda i. \{ \langle x, i \rangle \mid \text{ling } x \}$$

- ▶ And pronouns (where i_0 is the most recently introduced dref in i):

$$\mathbf{SHE}_0 = \lambda i. \{ \langle i_0, i \rangle \}$$

Binding

- ▶ Introducing drefs for thirsty pronouns can happen modularly:

$$m^\blacktriangleright = \text{do } x \leftarrow m \\ \lambda i. (\text{return } x) i+x$$

- ▶ Example of how this works for an indefinite:

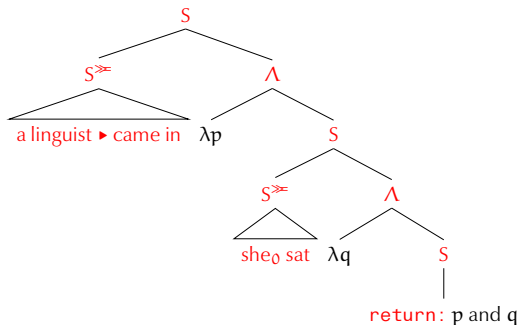
$$\mathbf{A.LING}^\blacktriangleright = \lambda i. \{ \langle x, i+x \rangle \mid \text{ling } x \}$$

- ▶ We can also \blacktriangleright -shift simple type e individuals injected into the monad with **return**:

$$(\text{return} : m)^\blacktriangleright = \lambda i. \{ \langle m, i+m \rangle \}$$

Dynamic binding via LF pied-piping

- ▶ Remarkably, rejiggering the semantics in this way predicts that dynamic binding arises via a kind of “LF pied-piping”:



- ▶ Unlike standard dynamic approaches, this derivation doesn't require a notion of dynamic conjunction.
 - ▶ In keeping with the approach I've been advocating, conjunction is boring and interacts with fancy things via `return` and \gg .

From another angle

- ▶ The “LF” from the last slide, in terms of do-notation:

```
do p ← (do x ← A.LING▶
        return: left x)
q ← (do y ← SHE0
     return: tired y)
return: p ∧ q
```

- ▶ The result here is equivalent to:

```
do x ← A.LING▶
  return: left x ∧ tired x
```

Closure

- ▶ Important part of any dynamic semantics: operators that **quantify over alternatives**. Usually: negation, things defined in terms of it.
- ▶ Negation, type $\text{Ft} \rightarrow \text{Ft}$:

$$\text{NOT} = \lambda m. \lambda i. \{ \{ \neg \exists \pi \in m \ i : \pi_0, i \} \}$$

- ▶ Universals, cf. $\neg \exists \neg \phi$, type $(e \rightarrow \text{Ft}) \rightarrow \text{Ft}$:

$$\text{EVERY.LING} = \lambda \kappa. \text{NOT} \left(\text{do } x \leftarrow \text{A.LING} \right. \\ \left. \text{NOT} (\kappa x) \right)$$

- ▶ Equivalent rendering of the universal:

$$\lambda \kappa. \lambda i. \{ \{ \forall x \in \text{ling} : \exists \pi \in \kappa x \ i : \pi_0, i \} \}$$

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Writer: the monad for supplemental content

- ▶ Giorgolo & Asudeh 2012 point out that the **Writer** monad is useful for modeling 2-dimensional content.
- ▶ Things in the Writer monad are **pairs** of values and some supplemental content:

$$F a = a \bullet t$$

- ▶ Where \bullet is just the pair constructor, i.e. $\langle \cdot, \cdot \rangle$. I use it to visually distinguish, and to emphasize the connection with Potts 2005.
- ▶ Injection is pairing a value with a trivial supplement. Sequencing involves *conjoining* supplemental content.

$$\begin{aligned} \text{return } x &:= x \bullet T & x \bullet p \gg \kappa &:= v \bullet p \wedge q \\ & & & \text{where } v \bullet q = \kappa x \end{aligned}$$

Combining Writer with dynamics

- ▶ A fancy \mathbf{a} can harbor nondeterminism, state-changing, and now, *supplemental content*:

$$\mathbf{F}\mathbf{a} ::= s \rightarrow \{\langle \mathbf{a} \bullet \mathbf{t}, s \rangle\}$$

- ▶ Monadic operations are expressed in terms of the “underlying” dynamic monad:

$$\begin{aligned} \text{return: } x &:: \text{return: } x \bullet \top & \mathbf{m} \gg \mathbf{k} &:: \text{do } x \bullet \mathbf{p} \leftarrow \mathbf{m} \\ & & & v \bullet \mathbf{q} \leftarrow \mathbf{k} x \\ & & & \text{return: } v \bullet \mathbf{p} \wedge \mathbf{q} \end{aligned}$$

- ▶ De-sugared version of sequencing:

$$\lambda i. \{ \langle v \bullet \mathbf{p} \wedge \mathbf{q}, h \rangle \mid \langle x \bullet \mathbf{p}, j \rangle \in \mathbf{m} i \wedge \langle v \bullet \mathbf{q}, h \rangle \in \mathbf{k} x j \}$$

Where we're headed

- ▶ John, who is a linguist (Fe):

return: $j \bullet \text{ling } j$

- ▶ A friend of mine, who is a linguist (Fe):

do $x \leftarrow \text{A.FRIEND}$

return: $x \bullet \text{ling } x$

- ▶ Sequencing any of these Fe's with the rest of the sentence:

do $x \leftarrow m$

return: left x

- ▶ No problem for supplemental things to interact with boring things.

Dynamic interactions

- ▶ Key bit: Writer + Dynamic monad is still a State monad!
- ▶ This means it is *totally kosher* to sequence something in the dynamic monad with a WriterT dynamic program:

```
do x ← m
do ..... ✓
```

- ▶ A trivial but revealing example – lifting a Fe into a Fe :

```
do x ← A.LING
return x =  $\lambda i. \{ \langle x \bullet T, i + x \rangle \mid \text{ling } x \}$ 
```

- ▶ Indeed, *any* Fa can be turned into a Fa , and any a into an Fa .

Comma

- ▶ Semantics for the comma intonation – turns a restrictive relative clause into a non-restrictive relative.

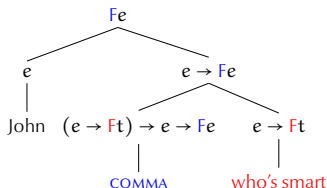
```
COMMA  $\kappa$  =  $\lambda x.$  do  $p \leftarrow \kappa x$   
return:  $x \bullet p$ 
```

- ▶ Type: $(e \rightarrow Ft) \rightarrow e \rightarrow Fe$
- ▶ De-sugared (notice that the type is $e \rightarrow Fe$):

$$\lambda x. \lambda i. \{ \langle x \bullet p, j \rangle \mid \langle p, j \rangle \in \kappa x i \}$$

Basic example

- ▶ A structure for *John, who's smart*:²



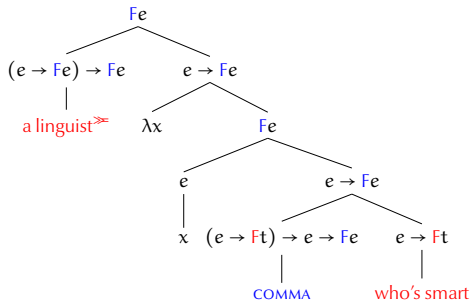
- ▶ Composing up via simple functional application, we end up with the following, as expected:

$$\begin{aligned} \text{COMMA } (\lambda x. \text{return} : \text{smart } x) j &= \lambda i. \{ \langle j \bullet \text{smart } j, i \rangle \} \\ &= \text{return} : j \bullet \text{smart } j \end{aligned}$$

²Suppressing derivation of the relative clause, but see Charlow 2014 for details.

Bridging the monads

- ▶ Can now fold in an indefinite, similarly:



- ▶ Yields the following meaning (again, composing by simple FA):

do $x \leftarrow \mathbf{A.LING}$
COMMA ($\lambda x.$ **return**: smart x) x

- ▶ Bottles up a nondeterministic value, supplement, updated state:

$= \lambda i. \{ \langle x \bullet \text{smart } x, i+x \rangle \mid \text{ling } x \}$

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Independence

- ▶ Why do universals, negation, etc. appear not to interact with appositive content?
- ▶ Their types are incompatible! E.g. universal needs to take scope over a $(e \rightarrow Ft) \rightarrow Ft$! It can't do anything with an $e \rightarrow Ft$.
- ▶ For exactly the same reason, it is impossible to anchor a NRRC to a universal. The types just don't fit.
- ▶ More generally, it appears that we don't require any meanings with negative occurrences of Fa types (save for "grammatical" operations like `return`, \gg , and perhaps λ).
 - ▶ This very closely mirrors the situation in Potts 2005.

Scope of the anchor

- ▶ Works for the same reason.
- ▶ Type of e.g. negation: Ft → Ft.
- ▶ There is just no way to combine this with an Ft. The negation doesn't know what to do with the extra dimension of content!
- ▶ But that's the only way for negation to scope over an indefinite-anchored NRRC! And that just won't work.

Binding into a NRRC

- ▶ A linguist knows John, who likes her:

```
do x ← A.LING ▶  
  do y ← λi. {⟨j • j likes i0, i⟩}  
  return: x knows y
```

- ▶ Evaluated and de-sugared:

$$\lambda i. \{ \langle x \text{ knows } j \bullet j \text{ likes } x, i + x \rangle \mid \text{ling } x \}$$

- ▶ The important bit: **A.LING** ▶ can bind into the appositive because *it makes sense* to sequence a **Fa** with a **Fa** (here, **Fe** and **Ft**).
- ▶ Again, the reason is that any **Fa** is a **Fa • t**.

Binding out of a NRRC

- ▶ John, who knows a linguist, likes her:

```
do x ← λi. {⟨j • j knows v, i + v⟩ | ling v}
y ← (do z ← HER0
      return: z)
return: x likes y
```

- ▶ Evaluated and de-sugared:

```
λi. {⟨j likes v • j knows v, i + v⟩ | ling v}
```

No binding for true quantifiers

- ▶ For, e.g., a universal to bind **into** an appositive, the universal would need to **take scope over** the appositive.
- ▶ Again, their types are incompatible. A universal needs to take scope over a $(e \rightarrow Ft) \rightarrow Ft$! It can't do anything with an $e \rightarrow Ft$.
- ▶ For, e.g., a universal to bind **out of** an appositive, you'd need the universal to be externally dynamic. Which it isn't:

$$\text{EVERY.LING} = \lambda\kappa. \text{NOT} \left(\text{do } x \leftarrow \text{A.LING} \right. \\ \left. \text{NOT} (\kappa x) \right)$$

Exceptional scope

- ▶ Since the theory I've proposed relies on **scope-taking**, you might expect that we have to appeal to exceptional scope-taking to explain cases when an NRRC occurs in an island.
- ▶ Well, yes and no. We do, but exceptional scope actually just *falls out of the monadic approach*. See Charlow 2014, 2015 for details.

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- ▶ Super natural to enrich the dynamic monad (nondeterminism and state) with supplemental content.
 - ▶ Involves nothing but the WriterT transform and a semantics for the comma intonation!
- ▶ Predicts a number of properties of NRRCs:
 - ▶ Independence/non-interaction
 - ▶ Which things can bind into and out of appositives, which can't
 - ▶ Scope of the anchor
- ▶ Fully compositional. No need for a representation language – direct model-theoretic interpretation.
 - ▶ Everything happens via functional application, with monadic combinators greasing the compositional skids.

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