September 12, 2014

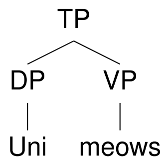
1 Today

Interpreting **intransitive** sentences like Uni meows, **transitive** sentences like Uni licked Porky, and even **ditransitive** sentences like Uni showed Puffy to Porky.

We'll start using sets, run into some roadblocks, then develop some tools for overcoming them. After we're done, we'll have the beginnings of a real, robust, *general* theory of interpretation.

1.1 Some test cases

Given what we learned in the last class, the intransitive case (Uni meows) seems easy. Just suppose meows denotes the set of meowers, and give a simple rule for interpretation in terms of **set membership**:



$$[\![\text{DP VP}]\!] = 1 \text{ iff } [\![\text{DP}]\!] \in [\![\text{VP}]\!]$$

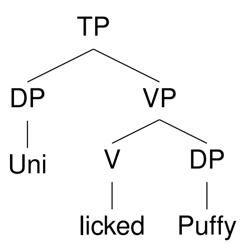
Here, iff Uni $\in \{x : x \text{ meows}\}$. (See H&K's long proof of a similar case.)

How about Uni meows and Uni purrs? OK: Another rule for interpretation:

$$[\![\text{TP}_1 \text{ and } \text{TP}_2]\!] = 1 \text{ iff } [\![\text{TP}_1]\!] = 1 \text{ and } [\![\text{TP}_2]\!] = 1$$

Here, iff Uni $\in \{x : x \text{ meows}\}$, and Uni $\in \{x : x \text{ purrs}\}$. Fine truth conditions, but they come at the price of another rule. What about disjunction?

Transitive cases like Uni licked Puffy are a harder nut to crack. Ideally, we would like to associate every node in the tree with an interpretation.



$$[\![\text{V DP}]\!] = ???$$

But we don't know how to associate the VP with a set, which would allow us to apply the earlier rule to form the whole sentence. Will we need another interpretation rule?

To say nothing of *ditransitive* cases like Uni showed Puffy to Porky. Will these require *yet another rule*??

1.2 No. Why our theory won't look this way

We'd *really* like to avoid case-by-case rules for how things are composed:

i. Don't really get any sense of how things work in the general case.
ii. I.e. not very explanatory.
iii. Super **syncategorematic**. Not everything gets assigned a meaning.

Can that really be how things work? We're gonna say NAH. What we would like is as *general* a characterization of the interpretation function $[\![\cdot]\!]$ as we can get.

What we will end up saying: meanings are either functions or arguments. Composing meanings is uniformly a process of *apply functions to arguments*.

We will need a bit of math to get there.

2 Some math

2.1 Start with a model

(See board)

Some facts about the model:

i. Some cats are meowing. Some aren't.
ii. Some cats are bigger than others.
iii. Some cats are to the left of others.

2.2 Relations

We can use relations to formalize features of this model.

Relations are sets of ordered pairs. The members of the ordered pairs stand in a certain relationship. What relation do you suppose the following sets might be?

i. $\{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,3\rangle, \langle 3,4\rangle, \cdots\}$

ii. {⟨Veneeta, semantics⟩, ⟨Simon, semantics⟩, ⟨Ken, syntax⟩, ⋯}

Diagram displays the licked relation (for convenience, the same as the bigger-than relation):

$$\{\langle \text{A}, \text{B}\rangle, \langle \text{A}, \text{C}\rangle, \langle \text{B}, \text{C}\rangle\}$$

Other ways of specifying this relation:

i. $\{\langle x, y\rangle : x \text{ licked } y\}$

ii. xRy iff x licked y

Relations may have certain properties (transitive, reflexive, antisymmetric, total, etc). It's good to know what these amount to, but we won't spend time on it.

2.3 Functions

We can use **functions** to talk about other features of the model.

A function f is any relation where each input is paired with at most one output. More formally, given any x, there is at most one $\langle x, y\rangle \in f$.

Instead of $\langle x, y\rangle \in f$, we'll write $f(x) = y$.

Functions **apply** to their arguments to give some value.

Functions (and relations) can be **partial**. Given some x, if a function (or relation) f doesn't include any pair of the form $\langle x, y\rangle$, then $f(x)$ is **undefined**.

Tabular notation for functions. Here, a partial square root function:

$$\begin{bmatrix} 1 \to 1 \\ 4 \to 2 \\ 9 \to 3 \end{bmatrix}$$

Here is a function that pairs each cat (that has a cat to its left) with the cat to its immediate left:

$$\begin{bmatrix} \text{B} \to \text{A} \\ \text{C} \to \text{B} \end{bmatrix}$$

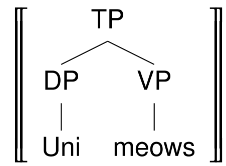
2.4 Characteristic functions

A function f is the **characteristic function** of a set S iff, for any x in S, $f(s) = 1$, and for any x not in S, $f(s) = 0$.

Example: call the characteristic function of the set of people in this room f. What is f(Veneeta)? What is f(Augustina)?

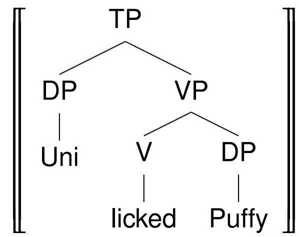
We can now rethink our entries for an intransitive verb like *meows*. Instead of denoting a set S, it'll denote the characteristic function on S.

Payoff: interpreting intransitives via functional application.



= ⟦meows⟧(⟦Uni⟧)

But what, now, about transitives? By analogy with the case we just did, we expect something like the following:



= ⟦licked Puffy⟧(⟦Uni⟧)

In other words, since VPs denote functions from individuals to truth values, so should licked puffy.

The rest of the class will be fleshing this out.

2.5 Functions into functions and Currying

Functions don't always have to return a value like 1 or 0. Functions might also return *other functions*.

Think about the addition operation. A natural way to think of it is as an operation that takes two numbers m and n at once and then gives you back $m + n$—i.e. as a function from pairs of numbers into a third number. But it could just as well take the things to be added *one at a time*!

The same goes for relations like the licked relation, the is-taller-than relation, etc. Each can be given in terms of a function, for example one that takes the licker and lick-ee one at a time.

First: you can just as well think of a relation in terms of the corresponding characteristic function, i.e. a function from pairs into 1 or 0:

$$\begin{bmatrix} \langle \mathrm{A},\mathrm{A}\rangle \to 0 \\ \langle \mathrm{A},\mathrm{B}\rangle \to 1 \\ \langle \mathrm{A},\mathrm{C}\rangle \to 1 \\ \langle \mathrm{B},\mathrm{A}\rangle \to 0 \\ \langle \mathrm{B},\mathrm{B}\rangle \to 0 \\ \langle \mathrm{B},\mathrm{C}\rangle \to 1 \\ \langle \mathrm{C},\mathrm{A}\rangle \to 0 \\ \langle \mathrm{C},\mathrm{B}\rangle \to 0 \\ \langle \mathrm{C},\mathrm{C}\rangle \to 0 \end{bmatrix}$$

From there it's a small step to **Currying/Schönfinkelization**: any n-ary relation can be turned into an n-place function. There are two ways to do this:

$$\begin{bmatrix} \mathrm{A} \to \begin{bmatrix} \mathrm{A} \to 0 \\ \mathrm{B} \to 1 \\ \mathrm{C} \to 1 \end{bmatrix} \\ \mathrm{B} \to \begin{bmatrix} \mathrm{A} \to 0 \\ \mathrm{B} \to 0 \\ \mathrm{C} \to 1 \end{bmatrix} \\ \mathrm{C} \to \begin{bmatrix} \mathrm{A} \to 0 \\ \mathrm{B} \to 0 \\ \mathrm{C} \to 0 \end{bmatrix} \end{bmatrix} \qquad \begin{bmatrix} \mathrm{A} \to \begin{bmatrix} \mathrm{A} \to 0 \\ \mathrm{B} \to 0 \\ \mathrm{C} \to 0 \end{bmatrix} \\ \mathrm{B} \to \begin{bmatrix} \mathrm{A} \to 1 \\ \mathrm{B} \to 0 \\ \mathrm{C} \to 0 \end{bmatrix} \\ \mathrm{C} \to \begin{bmatrix} \mathrm{A} \to 1 \\ \mathrm{B} \to 1 \\ \mathrm{C} \to 0 \end{bmatrix} \end{bmatrix}$$

Left-to-right $l \to r \to \langle l, r\rangle$ **Right-to-left** $r \to l \to \langle l, r\rangle$

The L position in a relation is (by convention) associated with subjects. The R position is (by convention) associated with objects. Thus:

i. Left-to-right Currying is faithful to the order of *terminals*.
ii. Right-to-left Currying is faithful to the order of *combination*.

Compositionality dictates #2.

3 The great payoff

A single rule for interpretation can get us everything we need today:

$$[\![X\ Y]\!] = [\![X]\!]([\![Y]\!]) \text{ or } [\![Y]\!]([\![X]\!]), \text{ whichever is defined}$$

3.1 Node by node compositionality

We can give meanings to every node in every tree we've considered up to now.

Intransitive verbs denote one-place characteristic functions:

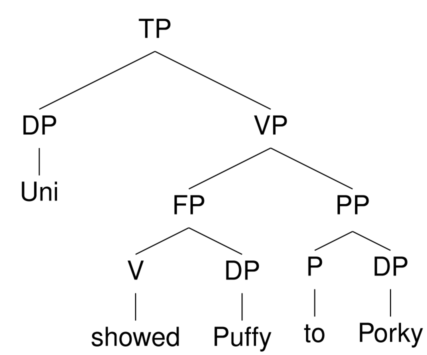
$$[\![\textsf{meow}]\!](x) = 1 \text{ iff } x \in \{y : y \text{ meows}\}$$

Transitive verbs denote two-place functions. Saturating one spot gives you something with the same sort of meaning as an intransitive.

$$[\![\textsf{licked}]\!](y)(x) = 1 \text{ iff } \langle x, y\rangle \in \{\langle x, y\rangle : x \text{ licked } y\}$$

Coordinators (*and*, *or*) denote two-place functions [left as an exercise].

Even a ditransitive(!): three-place functions. Saturating one spot gives you something like a transitive. Saturating two gives you something like an intransitive.



$$[\![\textsf{showed}]\!](z)(y)(x) = 1 \text{ iff } \langle x, z, y\rangle \in \{\langle x, z, y\rangle : x \text{ showed } z \text{ to } y\}$$

Food for thought: if *showed* is a 3-place relation on individuals (as below), what does this suggest about the meaning of *to*?

4 Next week

4.1 Mysteries remain

Determiners like *the*, *a*, and *every*. Interpreting quantified DPs (in certain configurations; the full story will come in the following weeks):

(1) Every dog licked Uni.

Pronouns:

(2) She meowed.

Relative clauses, adjectives, and both:

(3) The cat who licked Uni meowed.

(4) The black cat meowed.

(5) The black cat who licked Uni meowed.

4.2 Reading

H&K Chs. 4 and 5.

Review H&K Ch. 2 (on the lambda calculus)