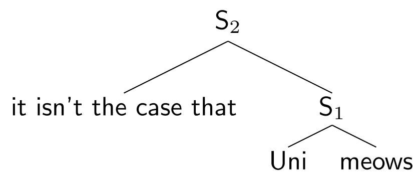
Solutions to Homework 1

Problem 1

- $[\![\textsf{it isn't the case that}]\!] = [\![\textsf{not}]\!] = \begin{bmatrix} 1 \to 0 \\ 0 \to 1 \end{bmatrix}$

- We begin with a structure to interpret and work top-down. Notice how every node is assigned an interpretation. $[\![\mathsf{S}_2]\!]$ expands into an application, $[\![\mathsf{S}_1]\!]$ expands into an application, and the terminal nodes (the leaves of the tree) are each associated with meanings.



$$
\begin{aligned}
[\![\mathsf{S}_2]\!] &= [\![\textsf{not}]\!]([\![\mathsf{S}_1]\!]) && \text{By the definition of } [\![\cdot]\!] \\
&= [\![\textsf{not}]\!]([\![\textsf{meows}]\!]([\![\textsf{Uni}]\!])) && \text{By the definition of } [\![\cdot]\!] \\
&= [\![\textsf{not}]\!]([\![\textsf{meows}]\!](\text{Uni})) && \text{Meaning of } \textsf{Uni} \\
&= [\![\textsf{not}]\!]((f : \text{for all } x,\ f(x) = 1 \text{ iff } x \in \{x : x \text{ meows}\})(\text{Uni})) && \text{Meaning of } \textsf{meows} \\
&= [\![\textsf{not}]\!](1 \text{ iff Uni} \in \{x : x \text{ meows}\}) && \text{Simplifying} \\
&= 0 \text{ iff Uni} \in \{x : x \text{ meows}\} && \text{Meaning of } \textsf{not} \\
&= 1 \text{ iff Uni} \notin \{x : x \text{ meows}\} && \text{Equivalent}
\end{aligned}
$$

Problem 2

- In terms of relations:

$$\textsf{and} : \{\langle l, r\rangle : l = r = 1\}$$

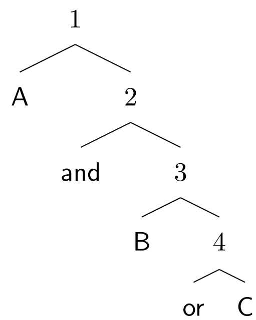
$$\textsf{or} : \{\langle l, r\rangle : l = 1,\ r = 1, \text{ or both}\}$$

In terms of Curry'd functions:

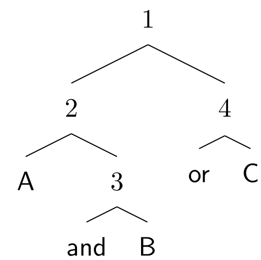
$$
\begin{bmatrix} 1 \to \begin{bmatrix} 1 \to 1 \\ 0 \to 0 \end{bmatrix} \\ \\ 0 \to \begin{bmatrix} 1 \to 0 \\ 0 \to 0 \end{bmatrix} \end{bmatrix}
\qquad
\begin{bmatrix} 1 \to \begin{bmatrix} 1 \to 1 \\ 0 \to 1 \end{bmatrix} \\ \\ 0 \to \begin{bmatrix} 1 \to 1 \\ 0 \to 0 \end{bmatrix} \end{bmatrix}
$$

$[\![\textsf{and}]\!]$ $\qquad$ $[\![\textsf{or}]\!]$

- We start with trees and go top-down. I leave out justifications of each step, but you should be able to reconstruct them. Notice again that the proof associates every node with an interpretation. Make sure you understand how to write $\llbracket \textsf{or} \rrbracket(1)$ and $\llbracket \textsf{and} \rrbracket(0)$ **as functions**.



$$
\begin{aligned}
\llbracket 1 \rrbracket &= \llbracket 2 \rrbracket(\llbracket \textsf{A} \rrbracket)\\
&= \llbracket \textsf{and} \rrbracket(\llbracket 3 \rrbracket)(\llbracket \textsf{A} \rrbracket)\\
&= \llbracket \textsf{and} \rrbracket(\llbracket 4 \rrbracket(\llbracket \textsf{B} \rrbracket))(\llbracket \textsf{A} \rrbracket)\\
&= \llbracket \textsf{and} \rrbracket(\llbracket \textsf{or} \rrbracket(\llbracket \textsf{C} \rrbracket)(\llbracket \textsf{B} \rrbracket))(\llbracket \textsf{A} \rrbracket)\\
&= \llbracket \textsf{and} \rrbracket(\llbracket \textsf{or} \rrbracket(1)(0))(0)\\
&= \llbracket \textsf{and} \rrbracket(1)(0)\\
&= 0
\end{aligned}
$$



$$
\begin{aligned}
\llbracket 1 \rrbracket &= \llbracket 4 \rrbracket(\llbracket 2 \rrbracket)\\
&= \llbracket \textsf{or} \rrbracket(\llbracket \textsf{C} \rrbracket)(\llbracket 2 \rrbracket)\\
&= \llbracket \textsf{or} \rrbracket(\llbracket \textsf{C} \rrbracket)(\llbracket 3 \rrbracket(\llbracket \textsf{A} \rrbracket))\\
&= \llbracket \textsf{or} \rrbracket(\llbracket \textsf{C} \rrbracket)(\llbracket \textsf{and} \rrbracket(\llbracket \textsf{B} \rrbracket)(\llbracket \textsf{A} \rrbracket))\\
&= \llbracket \textsf{or} \rrbracket(1)(\llbracket \textsf{and} \rrbracket(0)(0))\\
&= \llbracket \textsf{or} \rrbracket(1)(0)\\
&= 1
\end{aligned}
$$

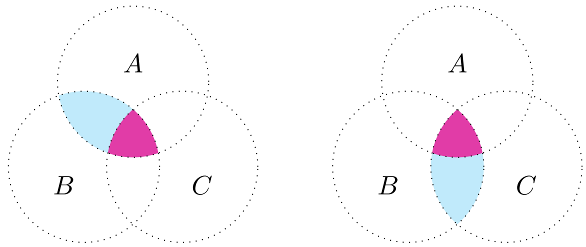
The structures give rise to distinct interpretations. This seems to me to do a decent job modeling the two possible meanings of the string A and B or C. Do you agree?

- The analysis **doesn't** extend to VP/DP/V coordination. Our semantics for and and or requires their arguments to be truth values. VPs, DPs, and verbs don't denote truth values. They denote (so far as we've seen) 1-place functions, individuals, and 2-place functions.

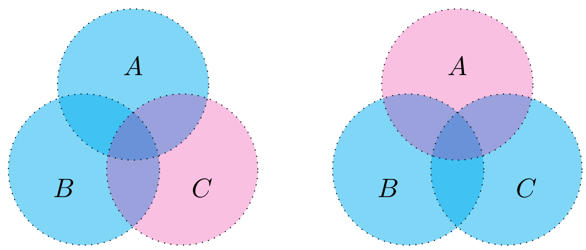
Problem 3

- Evaluate the following claims:
  - $\emptyset \in \{\emptyset\}$: True!
  - $\emptyset \subset \{\emptyset\}$: **Also** True! First, $\emptyset \subseteq \{\emptyset\}$ since everything in $\emptyset$ is in $\{\emptyset\}$. Second, $\{\emptyset\} \not\subseteq \emptyset$.
- $A \cup Z = A$ whenever $Z \subseteq A$.
- $A \cap Z = A$ whenever $A \subseteq Z$.

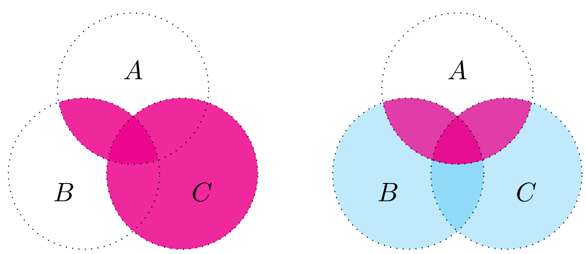
- Venn diagrams (where there is a dark magenta, take that to be the final set):



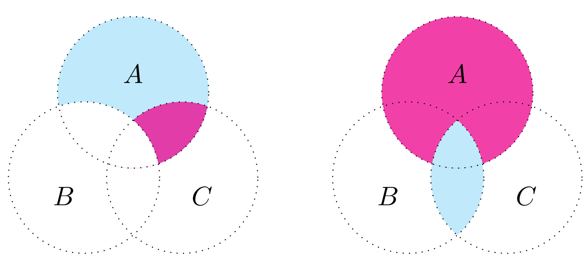
$(A \cap B) \cap C \quad = \quad A \cap (B \cap C)$



$(A \cup B) \cup C \quad = \quad A \cup (B \cup C)$



$(A \cap B) \cup C \quad \neq \quad A \cap (B \cup C)$



$(A - B) \cap C \quad \neq \quad A - (B \cap C)$

Problem 4

- Let J abbreviate *John comes*. Let B abbreviate *Bill comes*. Let S abbreviate *the party will be a success*.
- The speaker said *if J and B then S*. Call this utterance p.
- The speaker did **not** say *if J or B then S*. Call this would-be utterance p^+.

- p^+ is stronger than p: from *if J or B then S* you can conclude *if J and B then S*. (In class it was debated whether this holds *in the general case*, but it clearly holds here. Notice that this *or* is to be interpreted **inclusively**, i.e. as *at least one of J, B.*)

- Therefore (assuming p^+ was Relevant), Quantity and Quality force us to conclude the speaker does not believe p^+. (Otherwise, the speaker should have said p^+!)

- Therefore (assuming the speaker is an authority, i.e. opinionated and knowledgeable) we conclude that p^+ is false. That is, we conclude that it's false that John or Bill coming guarantees a successful party. Call this inference p^-. This is as far as I was expecting anyone to get in the homework. Feel free to stop reading here.

- Still with me? Ok. A question asked in class was: from the speaker's original utterance of p, we actually seem to conclude something that seems strictly **stronger** than p^-, namely that if just one of them comes, the party won't be a success.

- Here, we'll chalk this up to something called **Conditional Excluded Middle** (CEM essentially says that for any p and q, either p guarantees q, or p guarantees *not* q). We reason as follows:

	John and Bill both coming guarantees success.	(p)
	At least one coming doesn't guarantee success.	(p^-)
∴	Only John or only Bill coming doesn't guarantee success.	($p^\star$)

  Together with CEM, $p^\star$ entails that only John or only Bill coming guarantees a **lack of success**.[1]

- If you'd like to read more about reverse implicatures, I have some older notes from another class that I've uploaded to the course website. Feel free to peruse, or not.

[1] There is an interesting issue that comes up here: if we applied CEM to p^-, we'd derive something inconsistent with p (i.e. at least one coming guarantees a lack of success). Why do we actually apply it to $p^\star$? This actually has an answer in the modern literature on scalar implicature. I can give some pointers if folks are interested.