# Homework for Wednesday September 23, 2015

Please **type** your answers, but feel free to draw the diagrams by hand. I encourage you to work in groups, but please write up your answers individually.

## A. Sets

1. Evaluate the following claims:
   (a) $\emptyset \in \{\emptyset\}$
   (b) $\emptyset \subset \{\emptyset\}$
2. Give a general characterization of the sets $Z$ such that $A \cup Z = A$.
3. Give a general characterization of the sets $Z$ such that $A \cap Z = A$.
4. Using Venn diagrams, evaluate the following claims:
   (a) $(A \cap B) \cap C = A \cap (B \cap C)$
   (b) $(A \cup B) \cup C = A \cup (B \cup C)$
   (c) $(A \cap B) \cup C = A \cap (B \cup C)$
   (d) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
   (e) $(A - B) \cap C = A - (B \cap C)$

## B. Relations and functions

1. An **equivalence relation** is one that's reflexive, symmetric, and transitive.
   (a) Is $\{(x, y) : x \text{ is at least as old as } y\}$ an equivalence relation? Justify your answer.
   (b) Is $\{(x, y) : x \text{ is the same age as } y\}$ an equivalence relation? Justify your answer.
2. The **inverse** of a function $f$, written $f^{-1}$, can be defined as follows:

$$f^{-1} := \{(x, y) : (y, x) \in f\}$$

   (a) What is the inverse of the function $f(x) = x \times 5$?
   (b) What is the inverse of the identity function?
   (c) The formal definition of $f^{-1}$ given above does not actually guarantee that $f^{-1}$ is a function. For which choices of $f$ does $f^{-1}$ fail to be a function?
   (d) Assuming that $f$ and $f^{-1}$ are both functional, what do we know about $f^{-1}(f(x))$?
3. The **composition** of two functions $f$ and $g$, written $f \circ g$, is defined as follows:

$$(f \circ g)(x) := f(g(x))$$

   (a) If $f(y) :=$ $y$'s father, and $g(x) :=$ $x$'s mother, what is $(f \circ g)(x)$?
   (b) If $f$ and $f^{-1}$ are both functions, what is $(f^{-1} \circ f)(x)$?

## C. Reverse implicatures

1. A standard account of why *or* tends to be read exclusively goes as follows:

> The speaker $S$ said *p or q* instead of *p and q*. Because *p and q* is stronger than *p or q* (that is, *p and q* asymmetrically entails *p or q*), if $S$ is being cooperative and obeying Quantity, she must not believe *p and q*. □

You should be able to verify that this reasoning derives the natural exclusive reading of the following sentence (i.e., where I mean that I've read exactly one of those two books):

(a) I read 'Pride and Prejudice' or 'Great Expectations'.

However, the exclusive interpretation of disjunction appears to be absent in the following sentence:

(b) I didn't read 'Pride and Prejudice' or 'Great Expectations'.

That is, this sentence seems to mean that I didn't read *even one* of those two books, and not that I didn't read *exactly one* of those two books (since that would be true if I read both of them!). This suggests that the disjunction is being read inclusively—in other words, that the exclusivity implicature is not derived.

What's going on here? More specifically, which step in calculating the implicature for the positive version of the sentence (a) cannot be executed for the negative version of the sentence (b)?